

On Born-Infeld Gravity in Three Dimensions

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Abstract

In this paper we explore different aspects of three dimensional Born-Infeld as well as Born-Infeld-Chern-Simons gravity. We show that the models have AdS and AdS-wave vacuum solutions. Moreover we observe that although Born-Infeld-Chern-Simons gravity admits a logarithmic solution, Born-Infeld gravity does not, though it has a limiting logarithmic solution as we approach the critical point.

1 Introduction

In this paper we would like to study some features of recently proposed three dimensional gravitational Born-Infeld theory whose action is given by [1]

$$I = -\frac{4m^2}{\kappa^2} \int d^3x \sqrt{-\det g} \left[\sqrt{-\det \left(1 - \frac{1}{m^2} g^{-1} G \right)} - \left(1 + \frac{\Lambda}{2m^2} \right) \right], \quad (1.1)$$

where $\kappa^2 = 16\pi G_3$ and $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$, with $g_{\mu\nu}$ being the three dimensional metric. An interesting feature of this action is that upon expansion of the action in terms of the curvature we get cosmological Einstein-Hilbert action at first order, whereas at second order it leads to NMG action[2]. More interestingly at third order we get the R^3 terms which have been obtained in [3] by making use of the requirement that a holographic c-theorem is exist for the theory. More recently different aspects of Born-Infeld gravity has been studied in [4, 5].

This model together with TMG [6, 7] and NMG [2] have provided a framework to study three dimensional gravity with higher curvature terms which have massive graviton excitations. The hope is that these models can eventually help us to understand three dimensional quantum gravity. Actually due to the fact that three dimensional pure gravity with negative cosmological constant admits BTZ black hole solution [8], it is believed that the theory is non-trivial quantum mechanically even though at the classical level it has no propagating degrees of freedom.

It is worth mentioning that adding higher derivative terms to action will generically lead to instabilities due to the present of ghost-like modes. Nevertheless it is proposed [9] that TMG model could be a well defined quantum theory for a proper boundary condition and at a particular value of the parameters of the theory. We note, however, that the situation seems to be more complicated that it was thought at first place. In fact soon after the proposal [9] it was shown [10] that the linearized equations of motion of TMG at the critical point have a new solution. This now solution known as logarithmic solution which has the same asymptotic behavior as AdS wave solution has been first obtained in [11, 12]. Adding this mode will change the nature of the theory (for more discussions see *e.g.* [13, 14, 15, 16, 17, 18, 19, 20]).

The aim of the present paper is to explore the possibility of having logarithmic solution in Born-Infeld as well as Born-Infeld-Chern-Simons gravity. We will also study rotationally symmetric solutions in the model which could be thought of as extremal BTZ black holes. By making use of the entropy function formalism [21] we also evaluate the entropy of the solutions.

The paper is organized as follows. In order to fix our notations in section two we will review and extend the vacuum solutions of Born-Infeld gravity where we will also evaluate the entropy of the extremal black holes of model by making use the entropy function formalism. In section three we study AdS wave solution in Born-Infeld gravity where we show that, unlike NMG, the model does not admits logarithmic solution though it can be reached as a limiting solution. In section four we extend our study to the Born-Infeld-Chern-Simons theory whose action is given by Born-Infeld plus three dimensional gravitational Chern-Simons term. The last section is devoted to conclusions.

2 AdS and black hole solutions

In this section we will review and extend rotationally symmetric solutions of Born-Infeld theory which includes AdS_3 vacuum as well as solutions with $AdS_2 \times S^1$ symmetry. The later solution may be thought of as extremal black hole solution in the model.

To find the equations of motion of the model it is useful to expand the determinant in terms of trace by which the action (1.1) may be recast to the following form [22]

$$I = -\frac{4m^2}{\kappa^2} \int d^3x \sqrt{-\det g} F(R, K, S), \quad (2.1)$$

where¹

$$F(R, K, S) \equiv \sqrt{1 + \frac{1}{2m^2} \left(R - \frac{1}{m^2} K - \frac{1}{12m^4} S \right)} - \left(1 + \frac{\Lambda}{2m^2} \right), \quad (2.2)$$

with

$$K \equiv R_{\mu\nu} R^{\mu\nu} - \frac{1}{2} R^2, \quad S \equiv 8R^{\mu\nu} R_{\mu\alpha} R^\alpha{}_\nu - 6RR_{\mu\nu} R^{\mu\nu} + R^3. \quad (2.3)$$

Using this form of the action the equations of motion read [5]

$$\begin{aligned} -\frac{\kappa^2}{8m^2} T_{\mu\nu} = & -\frac{1}{2} F g_{\mu\nu} + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) F_R + F_R R_{\mu\nu} \\ & + \frac{1}{m^2} \{ 2\nabla_\alpha \nabla_\mu (F_R R^\alpha{}_\nu) - g_{\mu\nu} \nabla_\beta \nabla_\alpha (F_R R^{\alpha\beta}) - \square (F_R R_{\mu\nu}) - 2F_R R_\nu{}^\alpha R_{\mu\alpha} \\ & + g_{\mu\nu} \square (F_R R) - \nabla_\mu \nabla_\nu (F_R R) + F_R R R_{\mu\nu} \} \\ & - \frac{1}{2m^4} \{ 4F_R R^\rho{}_\mu R_{\rho\alpha} R^\alpha{}_\nu + 2g_{\mu\nu} \nabla_\alpha \nabla_\beta (F_R R^{\beta\rho} R^\alpha{}_\rho) + 2\square (F_R R_\nu{}^\rho R_{\mu\rho}) \\ & - 4\nabla_\alpha \nabla_\mu (F_R R_\nu{}^\rho R^\alpha{}_\rho) + 2\nabla_\alpha \nabla_\mu (F_R R R^\alpha{}_\nu) - g_{\mu\nu} \nabla_\alpha \nabla_\beta (F_R R R^{\alpha\beta}) \\ & - \square (F_R R R_{\mu\nu}) - 2F_R R R_\nu{}^\rho R_{\mu\rho} - g_{\mu\nu} \square (F_R R^2_{\alpha\beta}) + \nabla_\nu \nabla_\mu (F_R R^2_{\alpha\beta}) \\ & - F_R R^2_{\alpha\beta} R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \square (F_R R^2) - \frac{1}{2} \nabla_\mu \nabla_\nu (F_R R^2) + \frac{1}{2} F_R R^2 R_{\mu\nu} \} = 0, \end{aligned} \quad (2.4)$$

where

$$F_R = \frac{\partial F}{\partial R} = \frac{1}{4m^2} \left[F + \left(1 + \frac{\Lambda}{2m^2} \right) \right]^{-1}. \quad (2.5)$$

In general it is difficult to solve the above equations, though since we are interested in solutions with $AdS_2 \times S^1$ symmetry one may utilize the entropy function formalism [21]. An advantage to work with the entropy function formalism is that in this formalism not only one can find the solution but also the entropy of the corresponding solution can be evaluated.

To proceed we start from an ansatz for the metric with $AdS_2 \times S^1$ symmetry as follows

$$ds^2 = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 (dz + e r dt)^2. \quad (2.6)$$

¹In this paper we will only consider the case where $m > 0$ and $\Lambda < 0$.

Then the entropy function is defined by²

$$S = 2\pi(eq - f), \quad (2.7)$$

where $f = -\frac{8\pi m^2}{\kappa^2} \sqrt{-\det g} F(R, K, S)$, evaluated at the ansatz (2.6). To find the solution one needs to extremize the entropy function with respect to the parameters v_1, v_2 and e .

Using the ansatz (2.6) the entropy function (2.7) reads

$$f = -\frac{4\pi v_1 v_2^{1/2}}{\kappa^2} \left[\frac{1}{4m v_1^3} \sqrt{(-v_2 e^2 + 4m^2 v_1^2)(3v_2 e^2 - 4v_1 + 4m^2 v_1^2) - (\Lambda + 2m^2)} \right]. \quad (2.8)$$

Extremizing the entropy function with respect to v_1, v_2 and e we find three algebraic equations for the parameters³

$$\begin{aligned} E_1 &= (-3v_2^2 e^4 + 3v_1 v_2 e^2 - 2v_2 v_1^2 e^2 m^2 + 4v_1^3 m^2 - 8v_1^4 m^4) \\ &\quad + 2m v_1^3 \sqrt{(3v_2 e^2 - 4v_1 + 4m^2 v_1^2)(\Lambda + 2m^2)} = 0, \\ E_2 &= (3v_2^2 e^4 - 3v_1 v_2 e^2 - 3v_2 v_1^2 e^2 m^2 + 4v_1^3 m^2 - 4v_1^4 m^4) \\ &\quad + m v_1^3 \sqrt{(3v_2 e^2 - 4v_1 + 4m^2 v_1^2)(\Lambda + 2m^2)} = 0, \\ q &= \frac{\pi e v_2^{3/2}}{\kappa^2 v_1^2 m} \frac{(9v_2 e^2 - 8v_1 - 4m^2 v_1^2)}{\sqrt{(3v_2 e^2 - 4v_1 + 4m^2 v_1^2)(\Lambda + 2m^2)}}. \end{aligned} \quad (2.9)$$

To solve these equations we note that from the equations E_1 and E_2 we find

$$E_1 - 2E_2 = (v_2 e^2 - v_1)(9v_2 e^2 - 4v_1^2 m^2) = 0. \quad (2.10)$$

Therefore we get two different solutions given by

$$1) v_2 = \frac{v_1}{e^2}, \quad 2) v_2 = \frac{4m^2 v_1^2}{9e^2}. \quad (2.11)$$

In the first case using the equations of motion for v_1 and e one gets

$$v_1 = -\frac{m^2}{\Lambda(4m^2 + \Lambda)}, \quad e^2 = \frac{\pi}{\kappa^2 q m} \frac{\Lambda + 2m^2}{\sqrt{-\Lambda(4m^2 + \Lambda)}}. \quad (2.12)$$

Plugging these expressions into the entropy function (2.7) we can find the entropy of the corresponding extremal black hole solution

$$S = \frac{4\pi^2}{\kappa^2 m e} \frac{\Lambda + 2m^2}{\sqrt{-\Lambda(4m^2 + \Lambda)}}, \quad (2.13)$$

²If we compactify the solution into two dimensions the parameter e can be interpreted as electric field so that q is the electric charge. Moreover we assume that $e > 0$. Since we are considering the extremal black hole, from dual CFT point of view it means we are dealing with left moving sector. The right moving sector can also be studied by assuming $e < 0$. On the other hand from three dimensional point of view the parameter q has to be associated with the angular momentum of the extremal black hole. Or equivalently the temperature of the left moving sector may be given in terms on $1/e$.

³To get these equations we assume $-v_2 e^2 + 4m^2 v_1^2 > 0$. The case of $-v_2 e^2 + 4m^2 v_1^2 = 0$ will be discussed in the next section.

which is physically meaningful for $\Lambda + 2m^2 > 0$. As we will see this is, indeed, the case if we want the theory to support an AdS vacuum.

From this solution which is essentially a locally AdS_3 solution it is evident that the model admits an AdS_3 vacuum solution. In fact parameterizing the metric of AdS_3 solution by

$$ds^2 = \frac{l^2}{y^2}(dy^2 - 2dudv), \quad (2.14)$$

the radius of the AdS metric is found to be

$$l^2 = -\frac{4m^2}{\Lambda(4m^2 + \Lambda)}. \quad (2.15)$$

Alternatively one can invert the above equation to find the cosmological constant in terms of the radius of the AdS_3 solution⁴

$$\Lambda = -2m^2 \left(1 - \sqrt{1 - \frac{1}{m^2 l^2}} \right). \quad (2.16)$$

Note that to have a real cosmological constant we need to assume $m^2 l^2 > 1$ (see foot note 3). By making use of this expression it is interesting to rewrite the entropy of the black hole in terms of the AdS radius

$$S = \frac{\pi^2}{3} \frac{1}{2\pi e} \frac{3l}{2G} \sqrt{1 - \frac{1}{m^2 l^2}} = 2\pi \sqrt{\frac{q}{6}} \frac{3l}{2G} \sqrt{1 - \frac{1}{m^2 l^2}}, \quad (2.17)$$

which may be compared with the Cardy formula. Indeed using the c-extremization [23] one may identify the central charge of the dual CFT as follows [4, 5]

$$c_L = \frac{3l}{2G} \sqrt{1 - \frac{1}{m^2 l^2}}. \quad (2.18)$$

Using the Crday formula,

$$S = \frac{\pi^2}{3} T_L c_L = 2\pi \sqrt{\frac{c_L \bar{L}_0}{6}}, \quad (2.19)$$

we are led to the following identification

$$\bar{L}_0 = q, \quad T_L = \frac{2}{\pi} \sqrt{\frac{Gq}{l \sqrt{1 - \frac{1}{m^2 l^2}}}}. \quad (2.20)$$

The identification of the temperature may also be understood from the period of the compact direction in the ansatz (2.6).

On the other hand for the second solution one finds

$$v_1 = \frac{9}{8} \frac{188 m^4 - 324 \Lambda m^2 - 81 \Lambda^2 \pm 9 \sqrt{(9 \Lambda + 34 m^2)(9 \Lambda + 2 m^2)(\Lambda + 2 m^2)}}{m^2 (52 m^4 - 972 \Lambda m^2 - 243 \Lambda^2)},$$

⁴Note that from this expression we have $\Lambda + 2m^2 = 2m^2 \sqrt{1 - 1/m^2 l^2} > 0$, as we anticipated.

$$e^2 = -\frac{32\pi}{27\kappa^2 q} \frac{m^2 v_1^2}{\sqrt{\frac{4}{3}v_1^2 m^2 - v_1}} \quad (2.21)$$

that requires to have $9\Lambda + 2m^2 > 0$ which is stronger condition than we had in the previous case. Actually this solution may be interpreted as a warped black hole solution. It is easy to find the entropy of the corresponding black hole using the entropy function (2.7)

$$S_2 = \frac{16\pi^2 m v_1^2}{81\kappa^2 e \sqrt{\frac{4}{3}v_1^2 m^2 - v_1}} \left(64 m^3 v_1 - 60 m - 27(\Lambda + 2m^2) \sqrt{\frac{4}{3}v_1^2 m^2 - v_1} \right) \quad (2.22)$$

3 AdS wave solution and Log gravity

Having found the AdS vacuum solution, it is interesting to study AdS wave solution for Born-Infeld gravity. AdS wave solutions for TMG and NMG models have been studied in [11, 12, 24] where it was shown that at the critical value of the parameters the solution develops logarithmic behaviors. The same situation has also been obtained in bi-gravity [25]. In this section we will see that the situation is a little bit different for Born-Infeld gravity.

As we have seen in the previous section the model admits an AdS_3 vacuum solution which can be parametrized as

$$ds^2 = \frac{l^2}{y^2} (dy^2 - 2dudv), \quad \text{with} \quad l^2 = -\frac{4m^2}{\Lambda(\Lambda + 4m^2)}. \quad (3.1)$$

To proceed, based on this notation, we consider an ansatz for the AdS wave solution as follows

$$ds^2 = \frac{l^2}{y^2} [dy^2 - 2dvdu - G(u, y)du^2], \quad (3.2)$$

where $G(u, y)$ is an arbitrary function to be determined by equations of motion. Plugging this ansatz in the equations of motion (2.9) one finds that the radius, l , has the same expression as that for the AdS solution. Moreover the function G obeys the following differential equation

$$\frac{y^4 \frac{\partial^4 G}{\partial y^4} + 2y^3 \frac{\partial^3 G}{\partial y^3} - m^2 l^2 \left(y^2 \frac{\partial^2 G}{\partial y^2} - y \frac{\partial G}{\partial y} \right)}{y^2 l m \sqrt{m^2 l^2 - 1}} = 0. \quad (3.3)$$

One might naively think that the dominator of the above equation is redundant. Actually this is not the case. In fact the dominator plays a crucial rule especially at the critical point $m^2 l^2 = 1$. Moreover for large ml limit one expects that the above equation reduces to that in NMG model. Indeed evaluating the limit correctly we find that there are contributions from the dominator too. More precisely at large ml limit one finds

$$\frac{1}{y^2 m^2 l^2} \left[y^4 \frac{\partial^4 G}{\partial y^4} + 2y^3 \frac{\partial^3 G}{\partial y^3} - \frac{2m^2 l^2 + 1}{2} \left(y^2 \frac{\partial^2 G}{\partial y^2} - y \frac{\partial G}{\partial y} \right) \right] + \mathcal{O}\left(\frac{1}{m^4 l^4}\right) = 0, \quad (3.4)$$

which at leading order is exactly the equation we have in NMG model [24].

The standard solution for the above fourth order Euler-Fuchs differential equation is $G = y^\alpha$ where α satisfies the following equation

$$\alpha(\alpha - 2) [(\alpha - 1)^2 - m^2 l^2] = 0. \quad (3.5)$$

Thus the generic solution for AdS-wave is

$$G(u, y) = G_0(u) + G_2(u)y^2 + G_+(u) \left(\frac{y}{l}\right)^{1+m l} + G_-(u) \left(\frac{y}{l}\right)^{1-m l} \quad (3.6)$$

where G 's are arbitrary functions of retarded time u . Note that the first two terms can be eliminated by a coordinate transformation [12].

It is natural to look for a possibility of having multiplicities in the roots of the characteristic equation (3.5). In fact we see that the equation has a multiplicity in the roots for $m^2 l^2 = 1$. It is, however, important to note that at this point the dominator of the equation (3.3) is zero. Therefore we are not allowed to set $m^2 l^2 = 1$. Nevertheless it is natural to look for a possible limiting solution when $m^2 l^2 \rightarrow 1$. Indeed starting from the following ansatz

$$G(u, y) = \ln \left(\frac{y}{l}\right) \left[G_1(u) \left(\frac{y}{l}\right)^2 + G_2(u) \right], \quad (3.7)$$

and plugging it into the equation (3.3), one arrives at

$$\frac{2(G_1 l^2 - G_2 y^2)}{y^2 l^3 m} \sqrt{-1 + m^2 l^2}, \quad (3.8)$$

which is zero in the limit of $m^2 l^2 \rightarrow 1$. It is worth noting that although the limit is well defined the log solution is not a solution of the equations of motion and indeed it can be treated as a limiting solution. This is in contrast with what happens in TMG and NMG where the solution at the critical point is a solution of the equations of motion.

4 Adding Chern-Simons term

In this section we would like to extend the Born-Infeld gravity by adding a three dimensional gravitational Chern-Simons term to the Born-Infeld action. The gravitational Chern-Simons action is given by

$$I_{CS} = \frac{1}{2\kappa^2 \mu} \int d^3x \sqrt{-g} \epsilon^{\lambda\mu\nu} \left(\Gamma^\rho_{\lambda\sigma} \partial_\mu \Gamma^\sigma_{\rho\nu} + \frac{2}{3} \Gamma^\rho_{\lambda\sigma} \Gamma^\sigma_{\mu\tau} \Gamma^\tau_{\nu\rho} \right). \quad (4.1)$$

Adding this term to the action the equations of motion (2.9) will be corrected by the term $\frac{1}{\mu} C_{\mu\nu}$, with $C_{\mu\nu}$ being the Cotton tensor. On the other hand since the Cotton tensor is zero for and AdS_3 solution, the model still admits an AdS_3 vacuum solution which is indeed the same solution as we had in the previous section (3.1). It is then obvious that the black hole solutions of Born-Infeld gravity which are locally AdS, are still solutions of the modified model. Nevertheless since the modified model is a parity violating model the nature of the theory changes drastically. In particular the central charges of left and right moving sectors of the dual CFT are not equal.

To find the corresponding central charges using the c-extremization procedure, we recall that when the left and right central charges are not equal what the c-extremization procedure computes is the mean value of the central charges [23]. In other words applying the procedure for our case the result of the previous section (2.18) should be read as follows

$$\frac{c_L + c_R}{2} = \frac{3l}{2G} \sqrt{1 - \frac{1}{m^2 l^2}}. \quad (4.2)$$

On the other hand due to the present of the Chern-Simons term the whole action is diffeomorphism invariant up to a boundary term. From boundary theory point of view this shows itself in the gravitational anomaly in the dual CFT. Therefore the difference between the central charges is non-zero and in fact should be proportional to the coefficient of the Chern-Simons term. More precisely we have [26]

$$c_L - c_R = -\frac{3}{G\mu}. \quad (4.3)$$

Therefore from the equations (4.2) and (4.3) we get

$$c_L = \frac{3l}{2G} \left(\sqrt{1 - \frac{1}{m^2 l^2}} - \frac{1}{\mu l} \right), \quad c_R = \frac{3l}{2G} \left(\sqrt{1 - \frac{1}{m^2 l^2}} + \frac{1}{\mu l} \right). \quad (4.4)$$

Following the suggestion of [9] it is natural to see if there is any point in the moduli space of the parameters where the theory could be chiral. In fact looking that the central charge we observe that there is a possibility to set $c_L = 0$ which could lead to a consistent chiral theory with a proper boundary condition. Actually in this case we find a chiral line along which the left handed central charge vanishes⁵

$$\sqrt{1 - \frac{1}{m^2 l^2}} = \frac{1}{\mu l}. \quad (4.5)$$

Of course it is clear that for large $m^2 l^2$ limit the above expression reduces to that in NMG model.

Having had AdS vacuum it is worth looking for AdS wave in this model. In fact starting from the AdS wave ansatz (3.2) the equations of motion reduces to the following differential equation for function G

$$\frac{y^4 \frac{\partial^4 G}{\partial y^4} + \left(2 - \frac{m}{\mu} \sqrt{m^2 l^2 - 1}\right) y^3 \frac{\partial^3 G}{\partial y^3} - m^2 l^2 \left(y^2 \frac{\partial^2 G}{\partial y^2} - y \frac{\partial G}{\partial y}\right)}{y^2 l m \sqrt{m^2 l^2 - 1}} = 0. \quad (4.6)$$

In large $m^2 l^2$ limit the above equation may be expanded to get

$$\frac{1}{y^2 m^2 l^2} \left[y^4 \frac{\partial^4 G}{\partial y^4} + \left(2 - \frac{l m^2}{\mu}\right) y^3 \frac{\partial^3 G}{\partial y^3} - \frac{2 m^2 l^2 + 1}{2} \left(y^2 \frac{\partial^2 G}{\partial y^2} - y \frac{\partial G}{\partial y}\right) \right] + \mathcal{O}\left(\frac{1}{m^4 l^4}\right) = 0 \quad (4.7)$$

which at leading order coincides with that for generalized massive gravity [24]. Note that to get the correct expansion it was crucial to take into account the contributions of the dominator.

⁵If we allow μ to be also negative c_R can be set to zero too.

It is clear that the most general solution of the above differential equation is in the form of y^α with constant α satisfying the following characteristic equation

$$\alpha(\alpha - 2) \left[(\alpha - 1)^2 - (\alpha - 1) \frac{m}{\mu} \sqrt{m^2 l^2 - 1} - m^2 l^2 \right] = 0. \quad (4.8)$$

Therefore the generic solution is

$$\begin{aligned} G(u, y) = & G_+(u) \left(\frac{y}{l} \right)^{\frac{1}{2\mu} (2\mu + m\sqrt{m^2 l^2 - 1} + \sqrt{m^2(m^2 l^2 - 1) + 4\mu^2 m^2 l^2})} \\ & + G_-(u) \left(\frac{y}{l} \right)^{\frac{1}{2\mu} (2\mu + m\sqrt{m^2 l^2 - 1} - \sqrt{m^2(m^2 l^2 - 1) + 4\mu^2 m^2 l^2})}. \end{aligned} \quad (4.9)$$

To write the above solution we used the fact that the quadratic dependence of the solution can be eliminated by a coordinate transformation.

It is interesting to note that at the critical chiral line given by the equation (4.5) where $c_L = 0$ the characteristic equation degenerates leading to a new logarithmic solution

$$G(u, y) = G_1(u) \ln \left(\frac{y}{l} \right) + G_2(u) \left(\frac{y}{l} \right)^{m^2 l^2 + 1}. \quad (4.10)$$

We observe that although the Born-Infeld gravity does not support the log gravity solution adding Chern-Simons extends the range of parameters to accommodate a log gravity solution. We note also that although at $ml = 1$ the characteristic equation (4.8) has multiplicity in the roots, the model is only well defined for $ml > 1$ and we are not allowed to set $ml = 1$. Moreover, unlike the case considered in previous section, in the present case the limit of $ml \rightarrow 1$ is not a well defined too. This means that we do not have a well defined limiting logarithmic solution as well.

5 Conclusions

In this paper we have studied some aspects of recently proposed Born-Infeld gravity. A nice feature of the proposed action is that upon expanding the action in powers of the curvature at second order the theory reduces to NMG model and at third order the R^3 terms coincident with terms obtained by making use of the holographic c-theorem [3]. Indeed the action was constructed to have such a feature. It is important to note that to get Born-Infeld action one has to sum up infinite terms. Truncating the infinite sum to a summation of finite terms would drastically change the physical content of the theory⁶.

In particular NMG model has a critical point given by $m^2 l^2 = 1/2$ where the central charge of the dual CFT theory is zero and the model admits a new vacuum solution that is not asymptotically locally AdS_3 [24]. It is believed that at this point the theory is dual to a LCFT [31, 32]. Nevertheless when we are considering the Born-Infeld gravity there is nothing special at $m^2 l^2 = 1/2$. In other words summing up the infinite terms, somehow, resolves this point. On the other hand one might suspect that the effect of the infinite summation is just shifting the value of the critical point to $m^2 l^2 = 1$. We note, however, that this is not the case. Indeed we encounter an interesting effect in Born-Infeld gravity.

Actually the critical point where the central charge vanishes is a singular point in the moduli space of the parameters of the theory. In other words the point can only be reached

⁶Actually such an effect has been led to an interesting inflationary model [27, 28].

in the limit of $m^2 l^2 \rightarrow 1$. Taking into account that usually we get a logarithmic solution at the critical point, this means that the log gravity is not a solution of the model, though we can have it as a limiting solution. It would be interesting to explore the meaning of this effect from dual CFT description.

We have also seen that adding Chern-Simons terms to the Born-Infeld gravity the situation will change drastically. Although the $ml = 1$ point is still a singular point in the moduli space of the parameters, there is a critical chiral line where the left handed central charge vanishes and the solution develops a logarithmic term. From our experience of TMG and NMG model [29, 30, 31, 32] we would expect that the theory along this critical line is dual to a LCFT. It would also be interesting to understand this correspondence better.

In this paper we have studied Born-Infeld gravity with the assumption $m > 0, \Lambda < 0$. It is possible to relax this assumption to the cases where $m < 0$ or $m^2 < 0$. In these case it is possible to have other critical points. Moreover throughout our paper we have chosen a specific sign for the action. It is, however, possible to use another signs by modifying the action with some parameters, named by η and σ in [4]. Actually we have done all of our computations for generic η and σ . But we noticed that the most physical choice is $\eta = 1$ and $\sigma = -1$. Therefore we have presented our results only for this specific choice.

We note that the logarithmic solutions we have found in this paper provide gravity descriptions for logarithmic CFT's which might have application in condensed matter physics.

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